Name: ____ Date: _______________

Rolling for the Big One

Grace plays a game with her nephew, Tim, in which they try to build the largest seven-digit number possible with randomly rolled numbers. Each player has a game board as shown:

_ , _ _ _ , _ _ _

Players take turns rolling a 10-sided polyhedron, each side labeled with a different digit from 0 through 9. On his or her turn, the player writes the number that comes up on the roll in one space on his or her game board. Once the digit is written, it cannot be moved. So far in the game, Tim's game board is:

_ , 76_ , 412

Grace's game board is:

_ , _66, _30

On her fifth turn, Grace rolls a 7. She must decide where to place the 7. She thinks 7 is a pretty large number, but she wonders whether she might roll a 7, 8, or 9 on her two remaining rolls. She decides to use the probability of rolling a number at least as large as 7 on her next two rolls to help her choose where to place the 7 she just rolled on her fifth turn. If the probability of rolling a 7, 8, or 9 is greater than $\frac{1}{2}$, Grace will put the 7 in the hundred thousands place; otherwise, she will put it in the millions place. Grace needs to calculate the probability of rolling a 7, 8, or 9 on either the 6th or 7th roll, written as P(7, 8, or 9 on the 6th roll or 7, 8, or 9 on the 7th roll).

1. Grace finds:

   a. P(rolling a 7 on any single roll) = $\frac{1}{10}$. Why is this true?

   b. P(rolling a 7, 8, or 9 on any single roll) = $\frac{3}{10}$. Why is this true?

   c. P(rolling a number less than 7 on any single roll) = $\frac{7}{10}$. Why is this true?

   d. If P(A) = the probability of rolling a 7, 8, or 9 on the 6th roll, and

      P(B) = the probability of rolling a 7, 8, or 9 on the 7th roll, then

      P(A or B) is not the same as P(A) + P(B). Why not?

2. Is the outcome on the 7th roll independent or dependent of the result of the 6th roll? Why or why not?

3. What is P(both the 6th and 7th rolls are less than 7)?

4. What is P(rolling 7, 8, or 9 on 6th roll or 7, 8, or 9 on 7th roll)?

5. Where should Grace place the 7 she rolled on her 5th turn? Explain.

<table>
<tr><td>Patchwork</td><td colspan="2">Rubric</td></tr>
<tr><td>The core elements of performance required by this task are:
• Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model
Based on these, credit for specific aspects of performance should be assigned as follows</td><td>points</td><td>section points</td></tr>
<tr><td>1. Gives correct answers such as:
a. P(rolling a 7 on any single roll) = $\frac{1}{10}$ because there are 10 possible digits that could be rolled with equal likelihood, one of which is a 7.

b. P(rolling a 7, 8, or 9 on any single roll) = $\frac{3}{10}$, since any of 3 possibilities are designated from the 10 that are possible.

c. P(0, 1, 2, 3, 4, 5, 6) = $\frac{7}{10}$, representing 7 specified outcomes of 10 possible outcomes.

d. P(A or B) ≠ P(A) + P(B) because P(A) + P(B) counts the probability that a 7, 8, or 9 is rolled both times [P(A and B)] twice. So, P(A or B) = P(A) + P(B) – P(A and B)</td><td>1

1

1

1</td><td>4</td></tr>
<tr><td>2. Gives correct answer such as:

The outcome on the 7th roll is independent of the result of the 6th roll because the outcome on the 6th roll doesn't affect outcomes on the last roll. That is, since the outcome of rolling any number on the 6th roll is $\frac{1}{10}$, and the outcome of rolling any number on the 7th roll is $\frac{1}{10}$, they are separate events that do not depend on each other. Each roll carries the same possibilities as any other roll.</td><td>1</td><td>1</td></tr>
<tr><td>3 Gives correct answer such as:

P(both the 6th and 7th rolls are less than 7) = P(rolling a number less than 7 on 6th roll and rolling a number less than 7 on 7th roll) = P(rolling a number less than 7 on 6th roll) · P(rolling a number less than 7 on 7th roll) = $(0.7)(0.7) = 0.49$.</td><td>2</td><td>2</td></tr>
<tr><td>4. Gives correct answer such as:

Using the answer to question 1.d above, P(rolling 7, 8, or 9 on 6th roll or 7, 8, or 9 on 7th roll) = 1– P(both the 6th and 7th rolls are less than 7) = $1 - 0.49 = 0.51$.
Or, $\frac{3}{10} + \frac{3}{10} - \left(\frac{3}{10} \cdot \frac{3}{10}\right) = \left(\frac{6}{10} - \frac{9}{100}\right) = \frac{51}{100} = 0.51$</td><td>2</td><td>2</td></tr>
<tr><td>5. Gives correct answer such as:
Grace should place the 7 in the hundred thousands place since the probability that she will get a roll at least as high as this before the end of the game is 0.51, which is greater than $\frac{1}{2}$. That is, she is more likely than not to roll a 7, 8, or 9 on her 6th or 7th roll.</td><td>2</td><td>2</td></tr>
<tr><td>Total Points</td><td></td><td>11</td></tr>
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